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Fate of Three-Dimensional Black Holes Coupled to a Scalar Field and the Bekenstein-Hawking Entropy

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ABSTRACT

Three-dimensional black holes coupled to a self-interacting scalar field is considered. It is known that its statistical entropy a' la Strominger does *not* agree with the Bekenstein-Hawking (BH) entropy. However I show that, by a careful treatment of the vacuum state in the *canonical* ensemble with a fixed temperature, which is the same as that of the BTZ black hole without the scalar field, the BH entropy may be exactly produced by the Cardy's formula. I discuss its several implications, including the fate of black holes, no-scalar-hair theorems, stability, mirror black holes, and higher-order corrections to the entropy.

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1 Introduction

Three-dimensional gravity with a negative cosmological constant $\Lambda = -1/l^2$ has been considered as the most concrete example of the AdS/CFT correspondence, which states, roughly speaking,

$$Z_{\rm AdS} \sim Z_{\partial({\rm AdS})} \tag{1.1}$$

between the partition functions $Z_{\rm AdS}$ on the bulk AdS space and $Z_{\partial({\rm AdS})}$ on its boundary $\partial(AdS)$ [1]. Especially for the BTZ black hole [2], Strominger has shown that the correspondence (1.1) is "precise", from the knowledge of the asymptotic isometry group $SO(2,2)$, which generates the two copies of the Virasoro algebra with the central charges [3, 4, 5]

$$\bar{c} = c = \frac{3l}{2G} \ , \tag{1.2}$$

and the Cardy's formula [6] for the density of states

$$\ln\rho \sim 2\pi\sqrt{\frac{c(\Delta - c/24)}{6}} + 2\pi\sqrt{\frac{\bar{c}(\bar{\Delta} - \bar{c}/24)}{6}} \tag{1.3}$$

when the AdS_3 vacuum, which corresponds to the mass eigenvalue of $M = -1/8G$, is chosen [7]: $Z_{\rm BTZ} = Z_{\partial({\rm BTZ})} \sim \exp\{2\pi r_+/4G\}$. ($r_+$ is the radius of the outer event horizon.) From the frequent appearance of the BTZ black hole geometry in the many higher dimensional black holes in string theory, this result has been far reaching consequence in the higher dimensional AdS/CFT also [8].

Recently, the generalization of the Strominger's result to the black holes coupled to a self-interacting scalar field have been studied [9, 10, 11]. There it is found that the asymptotic symmetry group and the algebra of the canonical generators remain the same as in pure gravity even though the fall-off of the fields at spatial infinity is *slower* than that of pure gravity such as the symmetry generators acquire non-trivial pieces from the scalar field. However, strangely, it is known that the statistical entropy from the Cardy's formula does not agree with the Bekenstein-Hawking (BH) entropy

$$S_{\rm BH} = \frac{2\pi r_+}{4G}, \tag{1.4}$$

such as the usual AdS/CFT correspondence (1.1) is *not* favored; this has been first observed by Natsuume et al [9] and later by Henneaux et al [10] in a more general context; but no proper explanation or resolution has been provided so far.

In this paper, I address this issue and I show that by a careful treatment of the vacuum state in the *canonical* ensemble with a fixed temperature, which is the same as that of the BTZ black hole without the scalar field, the BH entropy is exactly produced by the Cardy's formula.

2 Asymptotic Symmetry of Three-Dimensional Gravity Coupled to a Scalar Field

In this section I review the result of Ref.[10] on the asymptotic symmetry of three-dimensional gravity minimally coupled to a scalar field. The associated action is given by

$$I = \frac{1}{\pi G}\int d^3x\sqrt{-g}\left[\frac{R}{16} - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]. \tag{2.1}$$

With the asymptotic conditions of the pure gravity [3], the scalar field does not contribute to the asymptotic symmetry generator if it decays rapidly enough, i.e., $\phi \sim r^{-(1+\varepsilon)}$ [3, 7, 9]. There exists, however, a more general form of the asymptotic conditions with a *slower* fall-off [10]

$$\phi = \frac{\chi}{r^{1/2}} - \alpha\frac{\chi^3}{r^{3/2}} + O(r^{-5/2}), \tag{2.2}$$

$$\begin{array}{ll} g_{rr} = \dfrac{l^2}{r^2} - \dfrac{4l^2\chi^2}{r^3} + O(r^{-4}), & g_{tt} = -\dfrac{r^2}{l^2} + O(1), \\ g_{tr} = O(r^{-2}), & g_{\varphi\varphi} = r^2 + O(1), \\ g_{\varphi r} = O(r^{-2}), & g_{t\varphi} = O(1), \end{array} \tag{2.3}$$

where $\chi = \chi(t,\varphi)$, and α is an arbitrary constant. The asymptotic isometry, which is generated by the following asymptotic Killing vectors

$$\begin{aligned} \xi^t &= l\left[T^+ + T^- + \frac{l^2}{2r^2}(\partial_+^2 T^+ + \partial_-^2 T^-)\right] + O(r^{-4}) , \\ \xi^r &= -r(\partial_+ T^+ + \partial_- T^-) + O(r^{-1}) , \\ \xi^\varphi &= T^+ - T^- - \frac{l^2}{2r^2}(\partial_+^2 T^+ - \partial_-^2 T^-) + O(r^{-4}) \end{aligned} \tag{2.4}$$

is the same as in pure gravity [3] $[2\partial_\pm = l\frac{\partial}{\partial t} \pm \frac{\partial}{\partial\phi}]$ and $T^\pm(\frac{t}{l} \pm \phi)$ generate two independent copies of the Virasoro algebra at the *classical* level

$$\begin{aligned} &\{L_m, L_n\} = i(m-n)L_{m+n} + \frac{ic}{12}m(m^2-1)\delta_{m+n,0}, \\ &\{\bar{L}_m, \bar{L}_n\} = i(m-n)\bar{L}_{m+n} + \frac{ic}{12}m(m^2-1)\delta_{m+n,0}, \\ &\{L_m, \bar{L}_n\} = 0 \end{aligned} \tag{2.5}$$

with the central charge (1.2) as in the pure gravity; this is a very non-trivial result since there are non-trivial contributions in the Virasoro generators $L_m, \bar{L}_m$ from the scalar field.

3 Black Holes with a Scalar Field

In this section, I consider the exact three-dimensional black hole solutions with a *regular* scalar hair of Henneaux-Martínez-Troncosoo-Zanelli (HMTZ) [10] and recall a few of its salient features for our considerations, especially on the decay of the black hole into the BTZ black hole, which has been considered recently by Gegenberg et al [11]. Further details can be founded in [10, 11].

Exact solutions for which the metric and the scalar field satisfy asymptotic conditions (2.2) and (2.3) can be obtained for a particular one-parameter family of potentials of the form

$$V_\nu(\phi) = -\frac{1}{8l^2}\left(\cosh^6\phi + \nu\sinh^6\phi\right) \ . \tag{3.1}$$

For $\nu > -1$, there is a solution that describes a static and circularly symmetric black hole, with a scalar field which is regular everywhere, given by [2]

$$\phi = \mathrm{arctanh}\sqrt{\frac{B}{H(r)+B}} \ , \tag{3.2}$$

where B is a non-negative integration constant

$$B = \sqrt{\frac{8Gl^2M}{3(1+\nu)}}, \tag{3.3}$$

for the black hole mass M, and

$$H(r) = \frac{1}{2}\left(r + \sqrt{r^2+4Br}\right) \ . \tag{3.4}$$

The metric reads

$$ds^2 = -\left(\frac{H}{H+B}\right)^2 F(r)dt^2 + \left(\frac{H+B}{H+2B}\right)^2 \frac{dr^2}{F(r)} + r^2 d\varphi^2 \tag{3.5}$$

with

$$F = \frac{H^2}{l^2} - (1+\nu)\left(\frac{3B^2}{l^2} + \frac{2B^3}{l^2H}\right) \ . \tag{3.6}$$

This HMTZ's solution satisfy the asymptotic conditions (2.2) and (2.3) with $\alpha = 2/3$. And the event horizon is located at

$$r_+ = l\Theta_\nu\sqrt{\frac{8GM}{3(1+\nu)}} \ , \tag{3.7}$$

[2]For $\nu = 0$ this solution reduces to the one found in the conformal frame ($\hat{g}_{\mu\nu} = (1-\hat{\phi}^2)^{-2}g_{\mu\nu}$, $\hat{\phi} = \tanh\phi$) [12].

where the constant Θ_ν is expressed, in terms of $z = 1 + i\sqrt{\nu}$, as [13]

$$\Theta_\nu = 2(z\bar{z})^{2/3}\frac{z^{2/3} - \bar{z}^{2/3}}{z - \bar{z}} \ . \tag{3.8}$$

The BH entropy and the Hawking temperature are given by, respectively,

$$S_{\rm BH} \ = \ \frac{2\pi r_+}{4G} = 2\pi l\sqrt{\frac{M}{2G}}\ \frac{\Theta_\nu}{\sqrt{3(1+\nu)}} \ , \tag{3.9}$$

$$T \ = \ \frac{\kappa}{2\pi} = \frac{3(1+\nu)}{2\pi l^2\Theta_\nu^2}\ r_+ \ . \tag{3.10}$$

Note that the specific heat $C = \partial M/\partial T = \pi r_+/2G$, which is exactly the same as that of the BTZ black hole with a vanishing scalar field, is positive such as the canonical ensemble with an equilibrium temperature with a heat bath exists. However, for a fixed temperature, apart from the above HMTZ black hole solution, the BTZ black hole can also be at equilibrium with the heat bath. This gives, from $T^{\rm HMTZ} = T^{\rm BTZ}$ for the Hawking temperature of the BTZ black hole $T^{\rm BTZ} = r_+^{\rm BTZ}/2\pi l^2$ [2], the following relationship between the horizon radii

$$r_+^{\rm HMTZ} = \frac{\Theta_\nu^2}{3(1+\nu)}\ r_+^{\rm BTZ} \ , \tag{3.11}$$

where $r_+^{\rm BTZ}$ is the horizon radius of the BTZ black hole [11]. In terms of the mass $M^{\rm HMTZ}$ of the HMTZ black hole, this becomes

$$\frac{M^{\rm HMTZ}}{M^{\rm BTZ}} = \frac{\Theta_\nu^2}{3(1+\nu)} < 1 \ , \tag{3.12}$$

where $M^{\rm BTZ} = r_+^2/(8Gl^2)$ is the mass of the BTZ black hole and the inequality comes from $\Theta_\nu^2 - 3(1+\nu) < 0$ for $\nu > -1$. This implies that there is a non-vanishing probability for the decay of the HMTZ black hole into the BTZ black hole, induced by the thermal fluctuations: The tunnelling amplitude is given by $\Gamma \approx Ae^{-\Delta I_E}$ [14] with some determinant A and the difference between the Euclidean actions of the higher and lower mass solutions at the *same temperature* $\Delta I_E = I_E[\bar{g}^{\rm BTZ}] - I[\bar{g}^{\rm HMTZ}] = \pi l \left[1 - \frac{3(1+\nu)}{\Theta_\nu^2}\right]\sqrt{\frac{M^{\rm HMTZ}\Theta_\nu^2}{6G(1+\nu)}} > 0$, where $\bar{g}$ is the classical solution of the Einstein equation.

4 Statistical Entropy

Once one has the Virasoro algebra of (2.5), one can compute its statistical entropy $S = \ln\rho$ for the density of states ρ as follows [6, 15, 16]

$$S = 2\pi\sqrt{\frac{c_{\rm eff}\Delta_{\rm eff}}{6}} + 2\pi\sqrt{\frac{\bar{c}_{\rm eff}\bar{\Delta}_{\rm eff}}{6}}, \tag{4.1}$$

where

$$c_{\rm eff} = c - 24\Delta_{\rm min}, \quad \Delta_{\rm eff} = \Delta - c/24,$$
$$\bar{c}_{\rm eff} = \bar{c} - 24\bar{\Delta}_{\rm min}, \quad \bar{\Delta}_{\rm eff} = \bar{\Delta} - \bar{c}/24 \ , \tag{4.2}$$

and $\Delta_{\rm min}(\bar{\Delta}_{\rm min})$ is the minimum of $L_0(\bar{L_0})$ eigenvalue $\Delta(\bar{\Delta})$; for $\Delta_{\rm min} = \bar{\Delta}_{\rm min} = 0$ this reduces to the formula (1.3), which is the case for the BTZ black hole. Then, by the usual relations [5, 17]

$$\Delta = \bar{\Delta} = \frac{lM}{2} + \frac{l}{16G} \tag{4.3}$$

and the central charge that exactly agrees with (1.2) of the BTZ black hole, one finds that $[\Delta_{\rm eff} = \bar{\Delta}_{\rm eff} = lM/2]$

$$S = 4\pi l \sqrt{-M_{\rm min} \ M} \ , \tag{4.4}$$

where $M_{\rm min}$ is the minimum eigenvalue of the HMTZ black hole mass M.

On the other hand, from the fact that, for a given Hawking temperature, the BTZ black hole is more stable thermodynamically than the HMTZ black hole, the true vacuum would be determined by that of the BTZ black hole; since the vacuum of the BTZ black hole is [3] $M^{\rm BTZ}_{\rm min} = -1/8G$, let the "corresponding" minimum mass eigenvalue $M^{\rm HMTZ}_{\rm min}$ for the HMTZ black hole is, by "assuming" the relation (3.12), given by

$$M^{\rm HMTZ}_{\rm min} = -\frac{1}{8G}\frac{\Theta^2_\nu}{3(1+\nu)} \ , \tag{4.5}$$

such as

$$c_{\rm eff} = \left(\frac{\Theta^2_\nu}{3(1+\nu)}\right)\frac{3l}{2G} \ . \tag{4.6}$$

Then, by plugging (4.5) into (4.4) one finally obtain the BH entropy (3.9) "exactly"; in the previous computations [9, 10] a naive vacuum of $M^{\rm HMTZ}_{\rm min} = -1/8G$, such as $\Delta_{\rm min} = \bar{\Delta}_{\rm min} = 0$, was considered and as a result, a wrong entropy $S = 2\pi l\sqrt{M/2G}$ was obtained; the true ground state of the HMTZ black hole is *lifted up* as $\Delta_{\rm min} = \bar{\Delta}_{\rm min} = (l/16G)(1 - \Theta^2_\nu/3(1+\nu))(> 0)$ by absorbing the scalar field.

5 Discussions

1. For $0 \leq \nu \leq 1$ some abnormal phenomena happen: Another horizon r_{++} which looks like a cosmological horizon appears outside the horizon r_+; It is created at infinity for $\nu = 0$; as ν

[3]There are some debates on this though. See for example [18, 19, 20, 21].

increases from 0, it approaches to r_+ until meets r_+ for $\nu = 1$; r_{++} is located inside of r_+ for $\nu > 1$ such as it behaves as an inner horizon. So, for $0 \leq \nu < 1$ the canonical ensemble with an equilibrium temperature does not exist, due to the different Hawking temperatures for the two horizons, unless they meet at $\nu = 1$. Therefore, for $0 \leq \nu < 1$ the black hole decay process in section 3 and the entropy computation in the section 4 do not apply.

2. Even though I obtained the BH entropy (3.9) precisely, by *assuming* (3.12) for the vacua with the negative mass eigenvalues $M^{\rm BTZ}_{\rm min} = -1/8G$ and $M^{\rm HMTZ}_{\rm min} = -\Theta^2_\nu/24G(1+\nu)$, the very meaning of (3.11) for the negative masses, where $r^{\rm HMTZ}_+$ and $r^{\rm BTZ}_+$ are pure-imaginary, is not quite clear. But from the exact agreement with the BH entropy, the assumed formula (3.12) must have some meaning. A possible explanation of this is to consider the black hole decay through the process '$HMTZ \rightarrow BTZ \rightarrow BTZ$ *vacuum*' and the proper value of $M^{\rm HMTZ}_{\rm min}$, which might not be an apparently acceptable state, may be extrapolated by the *analytic continuation* of (3.12) due to some reasons, such as (3.12) might have more general meaning than (3.11).

3. The Riemann tensor of the HMTZ metric (3.5) is singular at the origin as can be shown by computing the curvature invariant

$$R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} = \sqrt{\frac{2^{13}}{3}\frac{l(GM)^{5/2}}{\sqrt{\nu+1}r^5}} + O(r^{-4}) \tag{5.1}$$

near the origin. For $M > 0, \nu > -1$ this singularity is hidden inside the event horizon r_+. But, in contrast to the BTZ black hole, the particle states with $M < 0$ have the naked curvature singularity at the origin; even more, (5.1), as well as the metric (3.5), becomes imaginary as M grows negative. Moreover, one can easily check that there is *no* conical singularity at the origin regardless of the sign of M.

So, there are some qualitative differences between the AdS_3 vacuum of the BTZ black hole and the vacuum (4.5) of the HMTZ black hole: In contrast to the former, the latter has the naked curvatures singularity as well as the complex metric and scalar field. The complex-valuedness might not be a problem if one *assume* the decay into the AdS_3 vacuum [4]. However, it is unclear whether all the massive HMTZ black holes decay into their corresponding BTZ black holes with the masses given by the relation (3.12) before reaching the particle states with $M < 0$, or there are some remnant transient states such as one of them decays into the AdS_3 vacuum.

4. The potential $V_\nu(\phi)$ of (3.1) is unbounded below for $\nu > -1$, such as the weak energy condition (WEC) is violated for *outside* the horizon, and the scalar field excitation around $\phi = 0$ becomes "tachyonic" with the mass-squared $m^2 = -3/4l^2$. So, the HMTZ black hole

[4]This is not forbidden in principle since both vacua satisfy the classical Einstein equation, which is required in identifying the transition amplitude Γ, due to the absence of the conical singularities. But, in contrast to $M > 0$ case, the tunnelling *probability* becomes $|\Gamma|^2 \approx |A|^2$ due to the imaginary valuedness of ΔI_E.

departs importantly from the assumptions of the "no-scalar-hair" theorems [22, 23]; departs from the old theorems [22] due to violation of $\phi dV_\nu(\phi)/d\phi \geq 0$ and also due to the non-vanishing surface integral at infinity, from the slower fall-off (2.2); and departs from the new theorem [23] due to violation of the WEC. [5] However, the decay of the HMTZ into the BTZ realizes *dynamically* the no-scalar-hair theorems.

5. $V_\nu(\phi)$ satisfies the stability bound $m^2 \geq -1/l^2$ for the perturbations "on" AdS_3 [25]. [6] But this does not necessarily guarantees the stability of the HMTZ solution since this solution is not "globally" AdS_3: It is already known, due to Martínez, that $\nu = 0$ solution is *unstable* under linear perturbations of the metric [26]; I suspect that $\nu > 0$ solutions would be also unstable since this is "more" tachyonic than the $\nu = 0$ solution, which is unstable already; $-1 < \nu < 0$ solutions "might" be stable though it is still tachyonic; an explicit computation on this stability under linear perturbations would be certainly interesting.

6. There is the mirror black hole solution with $\tilde{M} = -M, 1+\tilde{\nu} = -(1+\nu) < 0$ ($\tilde{\nu} = \nu = -1$ solution corresponds to the massless black holes) with the metric $\tilde{g}_{\mu\nu} = g_{\mu\nu}$ and the scalar field $\tilde{\phi} = \phi$ $[\tilde{B} = B, \tilde{H} = H, \Theta_{\tilde{\nu}} = \Theta, \tilde{r}_+ = r_+, \tilde{T} = T]$ for

$$\tilde{V}_{\tilde{\nu}}(\tilde{\phi}) = -\frac{1}{8l^2}\left(\cosh^6\tilde{\phi} - (\tilde{\nu}+2)\sinh^6\tilde{\phi}\right) = V_\nu(\phi), \tag{5.2}$$

$$\tilde{F} = \frac{\tilde{H}^2}{l^2} + (1+\tilde{\nu})\left(\frac{3\tilde{B}^2}{l^2} + \frac{2\tilde{B}^3}{l^2\tilde{H}}\right) = F; \tag{5.3}$$

one can not distinguish this with the HMTZ black hole except for $\tilde{M} < 0, \tilde{\nu} < -1$. The role of this mirror solution to the vacuum decay into the AdS_3 vacuum is not clear.

7. The higher order corrections in the bulk, due to the thermal fluctuations of black hole geometry–fluctuations of the metric– to the BH entropy is given by, following [27],

$$S = S_{\rm BH} - \frac{3}{2}\ln S_{\rm BH} + \frac{1}{2}\ln\left[\frac{\pi^3 l^4(\delta E)^2}{2G^2}\left(\frac{3(1+\nu)}{\Theta_\nu^2}\right)^2\right]. \tag{5.4}$$

On the other hand, the corresponding correction to statistical entropy *a′ la* Strominger is given by

$$S = S_{\rm BH} - 3\ \ln S_{\rm BH} + \ln\left[\frac{64\pi^3 l^2}{(8G)^2}\frac{\Theta_\nu}{\sqrt{3(1+\nu)}}\right]. \tag{5.5}$$

This result shows the factor of 2 disagreement with the bulk result (5.4) as in the BTZ black hole without a scalar field [27, 28]. It would be interesting to compute the contribution due to

[5] This seems to be a generic feature of the hairy solutions. See [24].

[6] In the original proof [25], $d \geq 4$ is implicitly assumed. But this extends also to $d = 3$.

the fluctuation of the scalar field. It would be also interesting to compare this with that of the horizon holography *a′ la* Carlip [29, 30, 27, 31].

8. My result implies that the CFT for the HMTZ black hole has no $SL(2, \mathbf{C})$ invariant vacuum, which generates zero-eigenvalues for the generators Δ and $\bar{\Delta}$. This is hard to understand from standard CFTs [6]. But such an issue is always the case for the Strominger's method [18] and needs further investigation[7].

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